$\Rightarrow$ * $I_n = 1 - nJ_n \Rightarrow I_n = 1 - \dfrac{n^2 + ne^{-n\frac{\pi}{2}}}{n^2+1}$

$$\boxed{I_n = \frac{1 - n.e^{-n\frac{\pi}{2}}}{n^2+1}}$$

4) $I_n = f(n)$ avec $f(x) = \dfrac{1 - x.e^{-x\frac{\pi}{2}}}{x^2+1}$

$$\lim_{x \to +\infty} x.e^{-x\frac{\pi}{2}} = \lim_{X \to -\infty} -\frac{2}{\pi} X.e^{X} = 0$$

Posons $X = -x\dfrac{\pi}{2}$

D'où $\lim\limits_{x \to +\infty} f(x) = 0$

$\Rightarrow \lim\limits_{n \to +\infty} I_n = 0$

On a $I_n + nJ_n = 1 \Rightarrow J_n = \dfrac{1}{n} - \dfrac{I_n}{n}$

$\Rightarrow \lim\limits_{n \to +\infty} J_n = 0$

EX22: $n \in IN$

$$I_n = \int_{n\pi}^{(n+1)\pi} e^{-nx}.\sin x\, dx$$

1) Soit

$$\begin{cases} u(x) = e^{-nx} \to u'(x) = -n\, e^{-x} \\ v'(x) = \sin x \to v(x) = -\cos x \end{cases}$$

$$I_n = \left[-e^{-nx}\cos x\right]_{n\pi}^{(n+1)\pi} - n \int_{n\pi}^{(n+1)\pi} e^{-nx}.\cos dx$$

Posons : $J_n = \int_{n\pi}^{(n+1)\pi} e^{-nx}.\cos dx$

$$\begin{cases} u(x) = e^{-nx} \to u'(x) = -n\, e^{-x} \\ v'(x) = \cos x \to v(x) = \sin x \end{cases}$$

$J_n = \left[-e^{-nx}\sin x\right]_{n\pi}^{(n+1)\pi} + n\, I_n = n\, I_n$

Car $\sin(k\pi) = 0 \ \forall k \in Z$

D'où: $I_n = \left[-e^{-nx}\cos x\right]_{n\pi}^{(n+1)\pi} - n^2 I_n$

On a :

$\cos(n\pi) = (-1)^n$ et $\cos(n+1)\pi) = -(-1)^n$

Donc :

$I_n = e^{-n(n+1)\pi}.(-1)^n + e^{-n^2\pi}.(-1)^n - n^2 I_n$

$\Rightarrow I_n = (-1)^n e^{-n^2\pi}.\left(\dfrac{1 + e^{-n\pi}}{n^2+1}\right)$

2) $|I_n| = e^{-n^2\pi}.\left(\dfrac{1 + e^{-n\pi}}{n^2+1}\right)$

$\Rightarrow \lim\limits_{n \to +\infty} |I_n| = 0$. Donc $\lim\limits_{n \to +\infty} I_n = 0$

EX23

$n \in IN^*$, $m \in IN$:

$$I_{(n,m)} = \int_0^1 t^n (1-t)^m dt \quad ; \quad I_{(n,0)} = \frac{1}{n+1}$$

1) Soit $u'(t) = t^n \Rightarrow u(t) = \dfrac{t^n}{n+1}$

$v(t) = (1-t)^m \Rightarrow v'(t) = -m(1-t)^m$

$$I_{(n,m)} = \left[\frac{1}{n+1} t^n (1-t)^m\right]_0^1 + \frac{m}{n+1}\int_0^1 t^{n+1}(1-t)^{m-1} dt$$

$\Rightarrow I_{(n,m)} = \dfrac{m}{n+1} I_{(n+1,m-1)}$

2) d'après 1)

$\Rightarrow I_{(n,m)} = \dfrac{m}{n+1} I_{(n+1,m-1)}$

$I_{(n+1,m-1)} = \dfrac{m-1}{n+2} I_{(n+2,m-2)}$

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